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Local-field study of optical intersubband saturation in a parabolic quantum well under crossed magnetic and electric fields

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Abstract. On the basis of a microscopic scattering theory, for a parabolic quantum well system subjected to crossed magnetic and electric fields, the local field including the third-order nonlinearity is determined. Then the optical response is investigated. It turns out that the magnetic and electric fields produce notable changes in the optical properties.

1. Introduction

Recently, the study of optical properties associated with intersubband transitions of quantum wells (QW) under magnetic or/and electric fields has been of increasing theoretical and experimental interest; many novel physical properties have been predicted and observed [1–4]. In particular, in the case of QW under crossed magnetic and electric fields, there are significant modifications of the electrical states of the QW, due to the competing effects of the confining potential and the potential resulting from the applied magnetic and electric fields. Consequently, the optical properties have been changed significantly. This indicates potential applications in infrared detectors and modulators. As is well known, of prime importance for practical device application is knowledge of the range of the input optical density that yields a linear response. Ahn and Chuang [5], Seilmeier *et al* [6], and Julien *et al* [7] have studied the optical intersubband saturation response of GaAs QW experimentally and theoretically, and information on the relaxation has been deduced. To our knowledge, little has been done as regards investigating the intersubband saturation behaviour of QW under crossed magnetic and electric fields.

On the other hand, remotely doped parabolic quantum wells (PQW) offer the opportunity to exploit an interacting electron system in the transition region between two and three dimensions [8–10]. Very recently, we have studied the linear optical response of a PQW under crossed magnetic and electric fields [11]. Our study has shown that due to the combination of the two parabolic confining forces, namely, one that is the quantum confinement, and the other which arises from the magnetic and electric fields, changes in the optical properties are evident for different applied magnetic and electric fields. It is also found that in the case of a perfect PQW, the optical response is not sensitive to the applied electric field, although the electric field changes the electronic structure.

In this paper we investigate the intersubband saturation of a PQW under crossed magnetic and electric fields by employing a local-field approach which includes the third-order nonlinearity. In section 2 the electronic structure of the PQW is derived, and in

section 2.2 a microscopic local-field theory is applied to study the optical response of the PQW. In section 3 detailed numerical results are presented. Conclusions are drawn in section 4.

2. Theory

2.1. Electronic structure in the parabolic quantum well

In the following we consider a perfect infinitely high parabolic quantum well structure subjected to a crossed electric field $\mathbf{F} (= (0, 0, F))$ and magnetic field $\mathbf{B} (= (0, B, 0))$ and choose a vector potential $\mathbf{A} = (zB_0, 0, 0)$ to describe the applied DC magnetic field. Since in this case the momentum operators \mathbf{p}_x and \mathbf{p}_y commute with the magnetic Hamiltonian, and taking advantage of the translational invariance along the interfaces (the xy -plane), the single-particle wave function takes the form

$$\Psi(\mathbf{r}) = \frac{1}{2\pi} e^{i(\mathbf{k}_{\parallel} \cdot \mathbf{r})} \psi(k_x, z) \quad (1)$$

where $\mathbf{k}_{\parallel} = (k_x, k_y)$ indicates the in-plane wave vector of the electrons. Because of the result obtained in [8], that in an ideal PQW structure the many-body effects (direct and exchange Coulomb interactions) are negligible, in this paper we shall leave out the many-body effects. Thus, under the influence of the crossed magnetic and electric fields, $\psi(k_x, z)$ could be described by the one-dimensional Schrödinger equation

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi(k_x, z)}{dz^2} + [V(k_x, z) - \varepsilon(k_x)] \psi(k_x, z) = 0 \quad (2)$$

where

$$V(k_x, z) = \frac{1}{2} m \omega_p^2 (z - z_0)^2 + \frac{1}{2m} \left[\hbar^2 k_x^2 - \left(\frac{\hbar k_x \omega_c + eF}{\omega_p} \right)^2 \right] \quad (3)$$

$$z_0 = \frac{\hbar k_x \omega_c + eF}{m \omega_p^2} \quad (4)$$

with $\omega_p^2 = \omega_0^2 + \omega_c^2$, $\omega_c = eB/m$ being the cyclotron (Larmor) frequency, and ω_0 the oscillation frequency of the confining potential. The wave function $\psi(k_x, z)$ is a function of k_x as a result of the k_x -dependence of the total potential $V(k_x, z)$ appearing in equation (2). z_0 corresponds to the position of the cyclotron orbit centre in the absence of the parabolic confinement. It is clear that the potential minimum has been displaced by z_0 , and the potential $V(k_x, z)$ has also been moved by the last term in equation (3). After some manipulation, the eigenenergies in the infinitely high PQW are given analytically by

$$\varepsilon_n(k_x) = \hbar \omega_p \left(n + \frac{1}{2} \right) + \frac{1}{2m} \left[\hbar^2 k_x^2 - \left(\frac{\hbar k_x \omega_c + eF}{\omega_p} \right)^2 \right] \quad (n = 0, 1, 2, \dots) \quad (5)$$

and the wave functions have the following form:

$$\psi_m(z - z_0) = H_m(z - z_0) \exp[-(z - z_0)^2/2] \quad (6)$$

where the $H_m(z - z_0)$ are the Hermite polynomials. It is found from equation (5) that in the absence of the electric field the effective mass $m(B) = m(0) \omega_p^2 / \omega_0^2$. From the equations above it is clear that the applied magnetic and electric fields have cross-coupling effects on the total potential. Accordingly, the wave functions and eigenenergies are affected by the coupling effects. But the energy separation between levels remains the same, namely,

$\hbar\omega_p/2$, for the same magnetic field. This implies that in an ideal PQW, an electric field has the same effect on all of the states.

In the low-temperature limit ($T \rightarrow 0$), the electron density in the PQW is given by

$$n(z) = \frac{\sqrt{2m}}{\pi^2\hbar} \sum_n \int \vartheta[\varepsilon_F - \varepsilon_n(k_x)][\varepsilon_F - \varepsilon_n(k_x)]^{1/2} |\psi_n(k_x, z)|^2 dk_x \quad (7)$$

where ϑ is the Heaviside step function.

The Fermi energy, ε_F , is determined from the above equation by using the global charge-neutrality condition. Assuming that only the ground state is populated by electrons, from equation (7), one obtains

$$a_0 k_x^2 + a_1 k_x + a_2 \leq 0 \quad (8)$$

where $a_0 = \hbar^2 \omega x_0^2$, $a_1 = -2eF\hbar\omega_c$, and $a_2 = -[e^2 F^2 + 2m\omega_p^2(\varepsilon_F - \frac{1}{2}\hbar\omega_p)]$. Thus, k_{max} and k_{min} are determined. For real k , this requires

$$\varepsilon_F \geq \frac{1}{2}\hbar\omega_p - \frac{e^2 F^2}{2m\omega_0^2}. \quad (9)$$

It is apparent from equation (8) that due to the coupling effects of the crossed magnetic and electric fields described via the term a_1 , in general $k_{max} \neq k_{min}$.

2.2. The local field in the PQW structure

We now consider the situation in which an external p-polarized monochromatic (angular frequency ω) plane wave $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}^B(z)e^{i(q_{\parallel}r - \omega t)}$ pumps the PQW structure at an angle of incidence θ . The PQW is sandwiched by two semi-infinite isotropic media characterized by their dielectric constants. For simplicity we assume that the dielectric constants of the barriers are real and equal to ϵ^B . In accordance with the scattering theory, and employing the Green's function technique, the field $\mathbf{E}(z)$ in the PQW obeys the integral equation [12]

$$\mathbf{E}(z) = \mathbf{E}^B(z) - \int \mathbf{G}(z - z')[\mathbf{J}^{(1)}(z') + \mathbf{J}^{(3)}(z')] dz' \quad (10)$$

where $\mathbf{E}^B(z)$ is the external field, and $\mathbf{G}(z - z')$ is the appropriate Green's function, which is given [13] by

$$\begin{aligned} \mathbf{G}(z - z') &= \left(\frac{c}{\omega}\right)^2 \frac{\exp(iq_{\perp}|z - z'|)}{2iq_{\perp}} \begin{pmatrix} q_{\perp}^2 & q_{\perp}q_{\parallel} \operatorname{sgn}(z' - z) \\ q_{\perp}q_{\parallel} \operatorname{sgn}(z' - z) & q_{\parallel}^2 \end{pmatrix} \\ &+ \left(\frac{c}{\omega}\right)^2 \delta(z' - z) \mathbf{e}_z \times \mathbf{e}_z \end{aligned} \quad (11)$$

where c is the speed of light in the medium, sgn stands for signum, and δ indicates the Dirac function; $\mathbf{q} = (q_{\parallel}, q_{\perp})$ is the wave vector of the incident light in the scattering plane (the xz -plane), and \mathbf{e}_z is the unit vector along the z -direction. The electromagnetic propagating (radiation) properties ($z' \rightarrow z$) have been well described by the so-called direct term (the first term); also the self-field dynamics is included in the second term.

The induced linear current density is determined via nonlocal constitutive relations, and is given by

$$\mathbf{J}^{(1)}(z') = \int \boldsymbol{\sigma}(z', z'') \mathbf{E}(z'') dz'' = \int \mathbf{T}(k_x, z') \mathbf{F}(k_x) dk_x \quad (12)$$

where $\sigma(z', z'')$ is the nonlocal linear conductivity tensor (an explicit expression for it is given in reference [12]), and

$$\mathbf{T}(k_x, z') = \begin{pmatrix} \phi(k_x, z') & 0 \\ 0 & \Phi(k_x, z') \end{pmatrix} \quad (13)$$

$$\mathbf{F}(k_x) = \boldsymbol{\alpha}(k_x)\boldsymbol{\beta}(k_x) \quad (14)$$

where the elements of the tensor $\boldsymbol{\alpha}(k_x)$ are given by

$$\alpha_{xx}(k_x) = \frac{2^{3/2}\mu_0 e^2 \hbar}{\pi^2 \sqrt{m}} \frac{[\varepsilon_F - \varepsilon_1(k_x)]^{1/2}}{[\hbar(\omega + i/\tau)]^2 - [\varepsilon_{21}(k_x)]^2} k_x^2 \varepsilon_{21}(k_x) \quad (15)$$

$$\alpha_{xz}(k_x) = -\alpha_{zx}(k_x) = \frac{i\sqrt{2}\mu_0 e^2 \hbar^2 (\omega + i/\tau)}{\pi^2 \sqrt{m}} \frac{[\varepsilon_F - \varepsilon_1(k_x)]^{1/2}}{[\hbar(\omega + i/\tau)]^2 - [\varepsilon_{21}(k_x)]^2} k_x \quad (16)$$

$$\alpha_{zz}(k_x) = \frac{\mu_0 e^2 \hbar}{\sqrt{2}\pi^2 m^{3/2}} \frac{[\varepsilon_F - \varepsilon_1(k_x)]^{1/2}}{[\hbar(\omega + i/\tau)]^2 - [\varepsilon_{21}(k_x)]^2} \varepsilon_{21}(k_x). \quad (17)$$

The quantity $\boldsymbol{\beta}(k_x) = [\beta_x(k_x), \beta_z(k_x)]$ is defined as

$$\boldsymbol{\beta}(k_x) = \int \mathbf{T}(k_x, z'') \mathbf{E}(z'') dz'' \quad (18)$$

The induced third-order nonlinear current density entering in equation (10), $\mathbf{J}^{(3)}(z')$, can be derived from the density-matrix approach via $\mathbf{J}^{(3)}(\mathbf{r}, \omega) = \text{Tr}\{\rho_1^{(3)} \mathbf{j}_0\}$, $\rho_1^{(3)}$ being the third-order part of the density-matrix operator, and \mathbf{j}_0 being the free part of the current-density operator (for a full expression the reader is referred to reference [14]). Here in the case of a two-level system, in order to simplify the expression and without losing general features, we only consider the leading term, namely, the z -component, and it is given by

$$J_z^{(3)}(z') = \int S(k_x) [\varepsilon_F - \varepsilon_1(k_x)]^{1/2} |\beta_z(k_x)|^2 \beta_z(k_x) \Phi(k_x, z') dk_x \quad (19)$$

where

$$S(k_x) = -\frac{\sqrt{2m}\mu_0 e^4 \hbar^3}{4\pi^2 m^4 \omega^2} \left[\frac{1}{\hbar(\omega + i/\tau) - \varepsilon_{21}(k_x)} \right] \left[\left(\frac{\omega + i/\tau}{2\omega + i/\tau} + \frac{\omega + i/\tau}{i/\tau} \right) \right. \\ \left. \times \frac{1}{[\hbar(\omega + i/\tau)]^2 - [\varepsilon_{21}(k_x)]^2} + \frac{-\omega + i/\tau}{i/\tau} \frac{1}{[\hbar(-\omega + i/\tau)]^2 - [\varepsilon_{21}(k_x)]^2} \right] \quad (20)$$

where τ is introduced as the phenomenological relaxation time of the conduction electrons associated with the intersubband transitions, in order to account for the damping. One may notice in equation (19) that the light intensity dependence is introduced via $|\beta_z(k_x)|^2$. In the derivation performed above, we have assumed that only the ground state is partly populated, and introduced the *ansatz*

$$\varepsilon_{21}(k_x) = \varepsilon_2(k_x) - \varepsilon_1(k_x) \quad (21)$$

$$\phi(k_x, z) = \psi_2(k_x, z)\psi_1(k_x, z) \quad (22)$$

and

$$\Phi(k_x, z) = \psi_1(k_x, z) \frac{d\psi_2(k_x, z)}{dz} - \psi_2(k_x, z) \frac{d\psi_1(k_x, z)}{dz}. \quad (23)$$

One may see in section 3 that due to the presence of the crossed magnetic and electric fields, the cross-coupling between the x - and z -direction motion of the electrons, even in

the long-wavelength limit, is significant, whereas in the absence of a magnetic field the cross-coupling is dramatically decreased [14].

It is clear that in order to obtain the local field, we have to determine the only unknown quantity $\beta(k_x)$. To determine the unknown quantity $\beta(k_x)$, one has to return to equation (10), and multiplying equation (10) with $\mathbf{T}(k_x, z)$, then integrating the resulting equation over z , $\beta(k_x)$ is uniquely determined via the following integral equations:

$$\beta(k_x) = N(k_x) - \int \int \mathbf{T}(k_x, z) \mathbf{G}(z - z') [\mathbf{J}^{(1)}(z') + \mathbf{J}^{(3)}(z')] dz' dz \quad (24)$$

where

$$N(k_x) = [N_x(k_x), N_z(k_x)] = \int \mathbf{T}(k_x, z) \mathbf{E}^B(z) dz. \quad (25)$$

In order to simplify the numerical calculations without affecting the general physical features, one may utilize the fact that the thickness of the PQW $d \ll \lambda$, λ being the wavelength of the light used to pump the PQW system, and neglect the slow variation of the external field across the PQW—that is, one may set $\exp(iq_{\parallel}z) = 1$. Hence, one immediately realizes that

$$N_x(k_x) = \int \phi(k_x, z) dz E_x^B = 0. \quad (26)$$

One may make the expression for $\beta(k_x)$ even more concise by using the facts that, in the intersubband transitions, the z -component of the field is mainly responsible for the optical process, which is rapidly varying across the well, and that the x -component of the local field, which slowly varies across the PQW [13–15], approximates to

$$\beta_x(k_x) \simeq 0. \quad (27)$$

One finds that the only undetermined quantity, the z -component of $\beta(k_x)$, $\beta_z(k_x)$, is thus given by

$$\beta_z(k_x) - \int \kappa_{zz}(k_x, k'_x) [\alpha_{zz}(k'_x) + S(k_x) \sqrt{\varepsilon_F - \varepsilon_1(k'_x)} |\beta_z(k'_x)|^2] \beta_z(k'_x) dk'_x = N_z(k_x) \quad (28)$$

where

$$\kappa_{zz}(k_x, k'_x) = \int \int \Phi(k_x, z) G_{zz}(z - z') \Phi(k'_x, z') dz dz'. \quad (29)$$

Having determined the local field in the PQW, one can easily proceed with the calculation of the absorption coefficient by putting the observation points outside the PQW ($z \rightarrow \pm\infty$); one finds that the absorbance A_p is given by

$$A_p = 1 - |r_p|^2 - |t_p|^2 \quad (30)$$

where the Fresnel reflection (r_p) and transmission (t_p) coefficients of the PQW structure are given by

$$r_p = -E_x^{(<)} / E_x^B \quad (31)$$

and

$$t_p = 1 + E_x^{(>)} / E_x^B \quad (32)$$

with

$$\begin{aligned} E_x^{(<)} = -E_x^{(>)} &= \left(\frac{c}{\omega}\right)^2 \frac{q_{\parallel}}{2i} \int [\alpha_{zz}(k_x) + S(k_x) \sqrt{\varepsilon_F - \varepsilon_1(k_x)} |\beta_z(k_x)|^2] \beta_z(k_x) \\ &\times \int \Phi(k_x, z') dz' dk_x. \end{aligned} \quad (33)$$

Here we notice that neglecting the local-field effect implies that $\mathbf{E}(z) \equiv \mathbf{E}_B(z)$ in equation (10), or $\beta(k_x) \equiv \mathbf{N}(k_x)$. In the calculation of the optical absorption one can easily implement this by replacing $\beta_z(k_x)$ with $N_z(k_x)$ in equation (33), which cannot give a correct prediction of the optical absorption (e.g. the location of the resonant peak, the absorption in magnitude, as well as the shape of the spectrum). If one takes the external field as the local field, it can be easily seen from equation (33) that the resonant peak is exactly located at the electronic resonance ε_{21} due to α_{zz} and $S(k_x)$, which have a resonance at ε_{21} , and the local-field resonance, which is manifested via β_{zz} , is left out. Meanwhile, due to the self-dynamics, the actual location is blue shifted from the electronic resonance in energy [16]. From previous studies of the local field in quantum well structures [11–16] it has been found that for the intersubband transitions in which the z -component of the field is mainly responsible for the transitions, with increase of the z -component the local-field effects play a more important role in determining the optical response, owing to the enhanced induced linear ($\mathbf{J}^{(1)}(z')$) and nonlinear current densities ($\mathbf{J}^{(3)}(z')$) in equation (10)—whereas the x -component of the field, which is found to be slowly varying across the well [14], can be well approximated by $E_x(z) \simeq E_x^B$. Also a higher electron density means that more electrons can contribute to the intersubband transition process, which can lead to a bigger induced current density; in such a case the local field can give rise to bigger corrections to the optical response.

3. Numerical results and discussion

In the following we present detailed numerical calculations of the optical absorption of a GaAs/AlGaAs PQW in the presence of uniform crossed magnetic and electric fields. Here, for the sake of simplicity, we neglect the variation of the effective mass across the PQW (since the Al composition in the well region varies parabolically), and $m = 0.065m_0$ is used. Taking the ratio 60:40 for the band-edge discontinuity, the conduction barrier height is thus taken to be $V_c = 240$ meV. The oscillation frequency ω_0 of the PQW is determined via

$$\omega_0 = \frac{2}{d} \sqrt{\frac{2V_c}{m}} \quad (34)$$

where d is the well width at the top, and $V_c = 240$ meV. The other parameters used are: the dielectric constant $\epsilon^B = 13.1$, the doping concentration $7 \times 10^{15} \text{ m}^{-3}$, and the angle of incidence $\theta = 73^\circ$. As far as the relaxation process is concerned, we assume that the variations of the magnetic and electric fields do not affect the relaxation constant, and this is taken as $\tau = 1.0$ ps. The optical excitation intensity as related to the amplitude of the field, E_0 , is given by $I_0 = \frac{1}{2} \epsilon_0 c_0 \sqrt{\epsilon^B} |E_0|^2$.

Figure 1 shows the optical absorption spectra as a function of the incident energy for different magnetic fields, such as $B = 0, 10, 15$ T. In (a) the thickness d of the PQW is 400 Å, and in (b) $d = 800$ Å. We know that the optical response of the PQW is not sensitive to the electric field [9, 11], but in order to see the coupling between the applied magnetic and electric fields, here we keep the electric field set at $F = 10 \text{ kV cm}^{-1}$. It seems from figure 1(a) that when the light intensity $I_0 = 0$, with increase in the magnetic field, the peak of the absorption spectrum moves upwards in energy, and the amplitude is reduced. It is known that the peak corresponds to the local-field resonance. The upwards shift can be explained by the diamagnetic shift. From equation (5) it is clear that the energy separation is increased with the increase of the magnetic field. Thus the local-field resonance condition is changed. This is manifested by the upwards shift. The reduction of the magnitude of

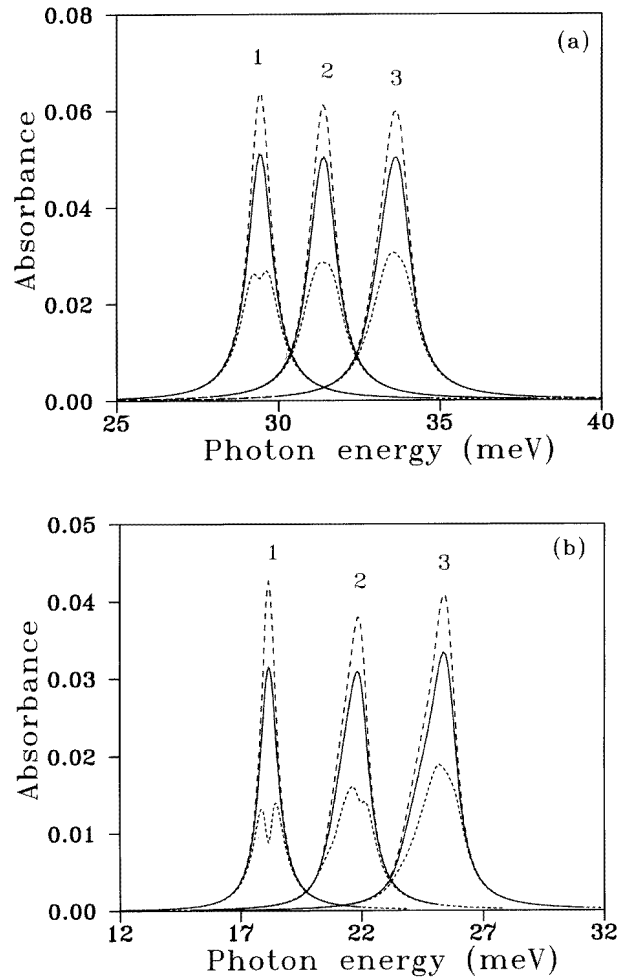


Figure 1. The absorbance as a function of incident energy for different magnetic fields, namely, $B = 0$ T (curve 1), 10 T (curve 2), and 15 T (curve 3). In (a), $d = 400$ Å and in (b) $d = 800$ Å. The electric field is $F = 10$ kV cm $^{-1}$, and the light intensity is 0 kW cm $^{-2}$ (long-dashed line), 2 kW cm $^{-2}$ (solid line), and 6 kW cm $^{-2}$ (short-dashed line), respectively. The relaxation time is $\tau = 1.0$ ps.

the absorption peak is due to the competing effects of the confining potential and the force resulting from the applied crossed magnetic and electric fields. One should note that in figure 1(a) the quantum confinement is strong ($\omega_0 > \omega_c$). As the magnetic field increases, the broadening of the absorption spectrum is mainly due to the coupling effect of the magnetic and electric fields. As the light intensity increases, $I_0 = 2$ kW cm $^{-2}$, the saturation of the absorption becomes evident. The location of the peak remains the same for the same magnetic field, and the amplitudes of the peaks are almost the same for different magnetic fields. When the intensity I_0 is increased further, say to $I_0 = 6$ kW cm $^{-2}$, a strong saturation occurs; for curve 1, two peaks appear because of the saturation [5]. We can see that the absorption is reduced by nearly half. It appears that upon increase of the applied magnetic field, the absorption becomes less sensitive to the light intensity.

In order to get a deeper insight into the saturation effect, we performed a calculation for a thicker PQW, $d = 800 \text{ \AA}$, the other parameters remaining the same. This is shown in figure 1(b). In this case, the quantum confinement is looser than in figure 1(a). In the limit $I_0 = 0$, for various magnetic fields, the amplitude of the absorption starts to decrease (0–10 T) and then increases (10–15 T) due to the competing effects. It is interesting to note that when $I_0 = 2 \text{ kW cm}^{-2}$, curve 3 becomes tilted. As the light intensity continues to increase, because of the strong saturation, the absorption spectrum is distorted, and the asymmetric shape becomes more apparent. It appears from a comparison with figure 1(a) that with the increase of the thickness of the PQW, the saturation effect becomes more pronounced.

4. Conclusions

On the basis of a microscopic local-field theory, the optical intersubband saturation of the GaAs/AlGaAs PQW structure subjected to crossed magnetic and electric fields has been investigated. It is shown that due to the presence of the magnetic and electric fields, the intersubband optical absorption peak is shifted upward with increase in the magnetic field; also, with increase in the thickness of the PQW, the PQW becomes more sensitive to the increase of the light intensity. We have shown that in accordance with the competition between the parabolic quantum confinement and that arising from the magnetic and electric fields, the optical absorption spectrum is changed. Our local-field calculations show that the absorption spectrum line is broadened with increase of the applied magnetic field, because the cross-coupling becomes stronger.

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